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Assignment II: MTH 213, Fall 2017

Ayman Badawi

QUESTION 1. (i) Let X be number of broken eggs. Given : a) X is an divisible by 3, b) $X \equiv 9 \pmod{10}$, and $X \equiv 6 \pmod{13}$.

a. Find X if $390 \leq X \leq 780$ [Note that X is divisible by 3 means $X \equiv 0 \pmod{3}$

b. Find X if $1560 \leq X \leq 1950$

(ii) Convert $(26)_7$ to base 2

(iii) Convert $(67012)_8$ to base 2

(iv) Convert $(117)_{16}$ to base 8

(v) Convert 93 to base 5

QUESTION 2 (HARD: I will discuss it on Thursday, it will not be on Quiz, note that this question is similar to question 1, 3 of Exam II Spring 2012). Solve over Z : $x \equiv 7 \ (mod\, 8)$, $3x \equiv 1 \ (mod\, 4)$, and $x \equiv 35 \ (mod\, 36)$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

Hanin Alrais:

1- (i) Given:

a- X is divisible by 3

b- $X \equiv 9 \pmod{10}$ and

$X \equiv 6 \pmod{13}$

a- Find X if $390 \leq X \leq 780$

b- Find X if $1560 \leq X \leq 1950$

CRT:

$$X \equiv 9 \pmod{10} \quad m_1$$
$$X \equiv 6 \pmod{13} \quad m_2$$
$$X \equiv 0 \pmod{3} \quad m_3$$

$gcd(m_1, m_2 m_3) = 1$

$x < m_1 m_2 m_3 \Rightarrow x < 390$

$\Rightarrow Q_1 = \frac{m}{m_1} = \frac{390}{10} = 39$

$y_1 = (Q_1 \bmod 10)^{-1} = (39 \bmod 10)^{-1}$

$= (9)^{-1}$

$9 \times \boxed{y_1} \bmod 10 = 1$

$\therefore y_1 = 9$

$\Rightarrow Q_2 = \frac{m}{m_2} = \frac{390}{13} = 30$

$y_2 = (Q_2 \bmod 13)^{-1} = (30 \bmod 13)^{-1}$

$= (4)^{-1}$

$4 \times \boxed{y_2} \bmod 13 = 1$

$\therefore y_2 = 10$

$X = (r_1 y_1 Q_1 + r_2 y_2 Q_2 + \cancel{r_3 y_3 Q_3}) \bmod 390$

$x = (9 \times 9 \times 39 + 6 \times 30 \times 10) \bmod 390$

$= (3159 + 1800) \bmod 390$

$= 4959 \bmod 390$

$= 279 \bmod 390$

$$\begin{array}{r|l} & 12 \\ \hline 390 & 4959 \\ & \underline{4680} \\ & 279 \end{array}$$

$\boxed{\text{all int.} \Rightarrow 279 + 390n}$

a- $279 + 390(1)$

$x = 669$

Chk → $390 \leq \underline{\underline{669}} \leq 780$ ✓

b- $279 + 390(4)$

$X = 1839$

Chk → $1560 \leq \underline{\underline{1839}} \leq 1950$ ✓

(ii) $(26)_7$ to base 2

conv. to base 10:

$2 \times 7^1 + 6 \times 7^0 = 14 + 6 = 20$

$(20)_{10}$ to base 2

$= (10100)_2$

(iii) $(67012)_8$ to base 2 $\qquad 2^{3} = 8$

$(110\ 111\ 000\ 001\ 010)_2$

(iv) $(117)_{16}$ to base 8

conv. to base 2

$\Rightarrow 0001\ 0001\ 0111$

conv. to base 8 "combine 3 digits"

$= (427)_8$

(v) 93 to base 5

Div 5	Mod 5
93 div 5 = 18	93 mod 5 = 3
18 div 5 = 3	18 mod 5 = 3
3 div 5 = 0 → stop.	3 mod 5 = 3

read

ans is $(333)_5$