

Assignment II: MTH 213, Fall 2017

Ayman Badawi

QUESTION 1. (i) Let X be number of broken eggs. Given : a) X is an divisible by 3, b) $X \equiv 9 \pmod{10}$, and $X \equiv 6 \pmod{13}$.

a. Find X if $390 \leq X \leq 780$ [Note that X is divisible by 3 means $X \equiv 0 \pmod{3}$]

b. Find X if $1560 \leq X \leq 1950$

(ii) Convert $(26)_7$ to base 2

(iii) Convert $(67012)_8$ to base 2

(iv) Convert $(117)_{16}$ to base 8

(v) Convert 93 to base 5

QUESTION 2 (HARD: I will discuss it on Thursday, it will not be on Quiz, note that this question is similar to question 1, 3 of Exam II Spring 2012). Solve over Z : $x \equiv 7 \pmod{8}$, $3x \equiv 1 \pmod{4}$, and $x \equiv 35 \pmod{36}$.

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1- (i) Given:

a- X is divisible by 3b- $X \equiv 9 \pmod{10}$ and $X \equiv 6 \pmod{13}$ a- Find X if $390 \leq X \leq 780$ b- Find X if $1560 \leq X \leq 1950$

CRT:

$$X \equiv 9 \pmod{10} \quad m_1$$

$$X \equiv 6 \pmod{13} \quad m_2$$

$$X \equiv 0 \pmod{3} \quad m_3$$

$$\gcd(m_1, m_2, m_3) = 1$$

$$X < m_1 m_2 m_3 \Rightarrow X < 390$$

$$\Rightarrow Q_1 = \frac{m}{m_1} = \frac{390}{10} = 39$$

$$y_1 = (Q_1 \pmod{10})^{-1} = (39 \pmod{10})^{-1}$$

$$= (9)^{-1}$$

$$9 \times \boxed{y_1} \pmod{10} = 1$$

$$\therefore y_1 = 9$$

$$\Rightarrow Q_2 = \frac{m}{m_2} = \frac{390}{13} = 30$$

$$y_2 = (Q_2 \pmod{13})^{-1} = (30 \pmod{13})^{-1}$$

$$= (4)^{-1}$$

$$4 \times \boxed{y_2} \pmod{13} = 1$$

$$\therefore y_2 = 10$$

$$X = (r_1 y_1 Q_1 + r_2 y_2 Q_2 + r_3 y_3 Q_3) \pmod{390}$$

$$\begin{aligned}
 x &= (9 \times 9 \times 39 + 6 \times 30 \times 10) \pmod{390} \\
 &= (3159 + 1800) \pmod{390} \\
 &= 4959 \pmod{390} \\
 &= 279 \pmod{390}
 \end{aligned}$$

$$\begin{array}{r}
 12 \\
 390 \overline{) 4959} \\
 \underline{4680} \\
 279
 \end{array}$$

$$\text{all int.} \Rightarrow 279 + 390n$$

a -

$$279 + 390(1)$$

$$x = 669$$

$$\text{chk} \rightarrow 390 \leq \underline{\underline{669}} \leq 780$$

$$\text{b - } 279 + 390(4)$$

$$x = 1839$$

$$\text{chk} \rightarrow 1560 \leq \underline{\underline{1839}} \leq 1950$$

(ii) $(26)_7$ to base 2

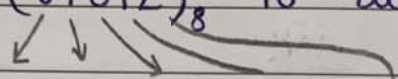
conv. to base 10:

$$2 \times 7^1 + 6 \times 7^0 = 14 + 6 = 20$$

$(20)_{10}$ to base 2

$$= (10100)_2$$

(iii) $(67012)_8$ to base 2 $2^3 = 8$



$$(110 \ 111 \ 000 \ 001 \ 010)_2$$

(iv) $(117)_{16}$ to base 8

conv. to base 2

$$\Rightarrow \underbrace{0001}_{1} \ \underbrace{0001}_{1} \ \underbrace{0111}_{7}$$

conv. to base 8 "combine 3 digits"

$$= (427)_8$$

(v) 93 to base 5

Div 5	Mod 5
93 div 5 = 18	93 mod 5 = 3
18 div 5 = 3	18 mod 5 = 3
3 div 5 = 0 \rightarrow stop.	3 mod 5 = 3

\uparrow Read

ans is $(333)_5$